D.N.

REPASO CALCULO INTEGRAL

SUMATORIAS

• Propiedades

* $\sum_{K=1}^{n} a^K = a^1 + a^2 + a^3 + a^4 + \ldots + a^n$

$\sum_{K=2}^{5} x^K = x^2 + x^3 + x^4 + x^5$ $\qquad$ $\sum_{K=1}^{3} K^2 = 1^2 + 2^2 + 2^3 = 14$

* $\sum_{K=1}^{n} C = Cn$, C: constante

$\sum_{K=1}^{3} = 15 = 3(15) = 45$ $\qquad$ $\sum_{K=1}^{4} 12 = 4(12) = 48$

* $\sum_{K=1}^{n} \left[a^K \pm b^K\right] = \sum_{K=1}^{n} a^K \pm \sum_{K=1}^{n} b^K$

$\sum_{K=1}^{3} \left[a^K + b^K\right] = \sum_{K=1}^{n} a^K + \sum_{K=1}^{n} b^K = \left[a^1 + a^2 + a^3\right] + \left[b^1 + b^2 + b^3\right]$

* $\sum_{K=1}^{n} \left[F(K) - F(K-1)\right] = F(n) - F(0)$

Sumatorias Telescopicas

$\sum_{K=1}^{200} \left[\sqrt{K} - \sqrt{K-1}\right] = F(n) - F(0) = \sqrt{200} - \sqrt{0} = 10\sqrt{2}$

* $\sum_{K=1}^{n} K = \frac{n(n+1)}{2}$

$\sum_{K=1}^{5} K = 1 + 2 + 3 + 4 + 5 = 15$ $\qquad$ $\sum_{K=1}^{5} K = \frac{5(5+1)}{2} = \frac{30}{2} = 15$

* $\sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{K=1}^{5} K^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$ $\qquad$ $\sum_{K=1}^{5} K^2 = \frac{5(5+1)(2(5)+1)}{6} = \frac{30 \cdot 11}{6} = 55$

* $\sum_{K=1}^{n} K^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\sum_{K=1}^{5} K^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$ $\qquad$ $\sum_{K=1}^{5} K^3 = [15]^2 = 255$

$$* \sum_{K=1}^{n} K^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$$

$$\sum_{K=1}^{5} K^4 = 1^4+2^4+3^4+4^4+5^4 = 979$$

$$\sum_{K=1}^{5} K^4 = \frac{5(6)[750+225+4]}{30} = \frac{30[979]}{30} = 979$$

INTEGRALES COMO SUMATORIAS

$$\lim_{n \to \infty} \sum_{K=1}^{n} f(x_K^*)(\Delta x)$$

$$\Delta x = \frac{b-a}{n} \quad ; \quad f(x_K^*) = \begin{cases} a+K(\Delta x) \\ 0^- \\ a+(K-1)(\Delta x) \end{cases}$$

$$\lim_{n \to \infty} \sum_{K=1}^{n} f[a+K(\Delta x)][\Delta x]$$ Extremo Izquierdo

0^-

$$\lim_{n \to \infty} \sum_{K=1}^{n} f[a+(K-1)(\Delta x)][\Delta x]$$ Extremo derecho

$$\int_{-1}^{2} 2x+3 \, dx$$

$$\lim_{n \to \infty} \sum_{K=1}^{n} f[a+(K-1)(\Delta x)][\Delta x]$$

$$\lim_{n \to \infty} \sum_{K=1}^{n} 2(a+(K-1)(\Delta x))+3 \, [\Delta x]$$

$$\lim_{n \to \infty} \sum_{K=1}^{n} (2[-1+(K-1)(3/n)]+3)[3/n]$$

$$\lim_{n \to \infty} \frac{3}{n} \sum_{K=1}^{n} \left[-2+2\left(\frac{3K-3}{n}\right)\right]+3$$

$$\lim_{n \to \infty} \frac{3}{n} \sum_{K=1}^{n} -2+\frac{6K}{n}-\frac{6}{n}+3$$

$$\lim_{n \to \infty} \frac{3}{n}\left[\sum_{K=1}^{n} 1 + \left(\frac{6}{n}\sum_{K=1}^{n} K\right) - \left(\frac{1}{n}\sum_{K=1}^{n} 6\right)\right.$$

$$\Delta x = \frac{b-a}{n} \qquad a = -1, \quad b = 2$$

$$\Delta x = \frac{2-(-1)}{n}$$

$$\Delta x = 3/n$$

$$\lim_{n\to\infty} \frac{3}{n}\left[n+\frac{6}{n}\cdot\frac{n(n+1)}{2}-\frac{1}{n}(6n)\right]$$

$$\lim_{n\to\infty} \frac{3}{\cancel{n}}\left[\cancel{n}+\frac{6}{n}\cdot\frac{\cancel{n}(n+1)}{2}-\frac{1}{n}6\cancel{n}\right]$$

$$\lim_{n\to\infty} 3\left[1+3\cdot\frac{n+1}{n}-\frac{6}{n}\right]$$

$$\lim_{n\to\infty} 3\left[1+3\cdot\left(\frac{n}{n}+\frac{1}{n}\right)\right]$$

$$\lim_{n\to\infty} 3\,[1+3(1)]$$

$$\lim_{n\to\infty} 3\,[4]$$

$$3\cdot 4$$

$$\boxed{12}$$

* Punto medio y aproximacion

$$\int_{-1}^{2} 2x+3\,dx$$

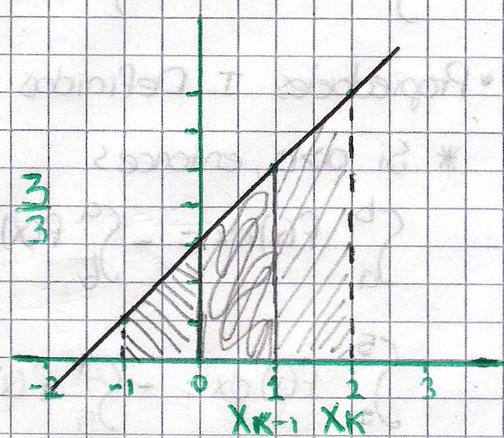
$$\sum_{k=1}^{n} f(\bar{x}_k)\,(\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{3}{3}$$

$$\Delta x = 1$$

n = # de intervalos = 3

$$\bar{x}_k = \frac{x_k + x_{k-1}}{2} = \frac{2+1}{2} = \frac{3}{2}$$



$$\sum_{k=1}^{3}\left[2\left(a+\frac{3}{2}(\Delta x)\right)+3\right][\Delta x]$$

$$\sum_{k=1}^{3}\left[2\left((-1)+\frac{3}{2}(1)\right)+3\right][1]$$

$$\sum_{K=1}^{3} [-2+3]+3$$

$$\sum_{K=1}^{3} 1+3 = \sum_{K=1}^{3} 4 = \boxed{12}$$

Nota: Cuando se tienen parabolas y se busca una mayor exactitud se aumenta el valor de n.

INTEGRALES

- Antiderivada

$$F(x) = \frac{x^{n+1}}{n+1} + C$$

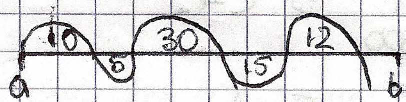
- Propiedades generales

* $\int dx = x + C$; $\int K dx = Kx + C$

* $\int f(x) \pm g(x)\, dx = \int f(x) \pm \int g(x)$

* $\int K f(x)\, dx = K \int f(x)\, dx$

La integral como Area
||
Areas encima del eje x menos Areas debajo del eje x



$= (10+30+12) - (5+15)$

$= 52 - 20$

$= 32$

- Propiedades I. Definidas

* Si a>b, entonces

$$\int_a^b f(x)dx = -\int_b^a f(x)\,dx$$

$$\int_7^5 f(x)\,dx = -\int_5^7 f(x)\,dx$$

* Si f(a) existe

$$\int_a^a f(x)\,dx = 0$$

$$\int_5^5 f(x)\,dx = 0$$

* Si K es una constante cualquiera

$$\int_a^b K dx = K(b-a)$$

$$\int_2^9 15\,dx = 15(9-2) = 105$$

* $\int_a^b f(x)\,dx = \int_a^c f(x) + \int_c^b f(x)$

$$\int_{e^x}^{\operatorname{sen} x} = \int_{e^x}^{0} + \int_{0}^{\operatorname{sen} x}$$

$$*\int_a^b m\,dx \leq \int_a^b f(x)\,dx \leq M\,dx \iff m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$*\int_a^x f(t)\,dt \underset{\text{TFC.}}{=} f(x)$$

$$\int_a^x \sqrt{1-t^3}\,dt = \sqrt{1-x^3}$$

- Redefiniendo una integral indefinida

$$*\int f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)(\Delta x) \iff \int_0^1 f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^4}{n^5} \iff \int_0^1 f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k^4}{n^5} \cdot n\right)$$

$$\text{Sea } x = \frac{i}{n} \rightarrow i = xn$$

$$f(x) = \frac{i^4}{n^4} = \frac{(nx)^4}{n^4} = x^4$$

$$\int_0^1 \lim_{n\to\infty} \sum_{k=1}^{n} x^4\,dx = \frac{x^5}{5}\Big|_0^1 = \frac{1}{5} - 0 = \boxed{\frac{1}{5}}$$

TEOREMA FUNDAMENTAL DEL CALCULO

- Primera parte

$$*\int_a^x f(t)\,dt = f(x)$$

$$\int_a^x \sqrt{1-t^3}\,dt = \sqrt{1-x^3}$$

$$*\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\frac{du}{dx}$$

$$\frac{d}{dx}\int_1^{x^4} \sec t\,dt = \frac{d}{dx}\int_1^{u} \sec t\,dt$$

$$= \frac{d}{du}\left[\int_1^u \sec(t)\,dt\right]\frac{du}{dx}$$

$$= \sec(u)\frac{du}{dx} = \boxed{\sec(x^4)\,4x^3}$$

- Segunda parte

$$\int_a^b f'(x) = f(b) - f(a)$$

$$\int_2^{-1} x^4 - 4x + 3 = \frac{x^5}{5} - \frac{4x^2}{2} + 3x$$

$$= \frac{x^5}{5} - 2x^2 + 3x \Big]_2^{-1}$$

$$= \frac{(-1)^5}{5} - 2(-1)^2 + 3(-1) - \left[\frac{(2)^5}{5} - 2(2)^2 + 3(2)\right]$$

$$= -\frac{1}{5} - 5 - \left[\frac{32}{5} - 2\right]$$

$$= -\frac{1}{5} - \frac{5}{1} - \frac{32}{5} + \frac{2}{1}$$

$$= \frac{-1 - 25 - 32 + 10}{5}$$

$$= \boxed{-48/5}$$

INTEGRALES POR SOSTITUCION

- $\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$

1. Denina cual sera la sustitucion "u" [g(x)]
2. Derive "u" = [g'(x) dx] "du"
3. Analice coincidencias. despeje "dx" y semejantes para remplazar
4. Remplace
5. Integre
6. Sustituya

$$\int (x^3+4)^5 \cdot x^2\, dx \quad \Big| \quad \int u^5 \frac{du}{3} = \frac{1}{3}\int u^5 du = \frac{1}{3}\frac{u^6}{6} = \frac{u^6}{18} + C$$

$u = x^3 + 4 \quad du = 3x^2 dx$

$x^2 dx = \frac{du}{3}$

$$\boxed{\frac{(x^3+4)^6}{18} + C}$$

• $\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

1. Defina cual sera la sostitucion "U" [g(x)]
2. Derive "U" = [g'(x) dx] "du"
3. Analice coincidencias despeje "dx" y semejantes para remplazar
4. Halle los valores de g(b) y g(a)
5. Remplace
6. Integre
7. Utilice T.F.C. y evalue la antiderivada en g(b) y g(a)

$\int_0^4 \sqrt{2x+1}\,dx$

$u = 2x+1 \qquad du = 2\,dx \qquad dx = \frac{du}{2}$

$g(b) = 2(4)+1 = 9$

$g(0) = 2(0)+1 = 1$

$\int_1^9 \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int_1^9 \sqrt{u}\,du = \frac{1}{2}\int_1^9 u^{1/2} = \frac{1}{2}\,\frac{u^{3/2}}{3/2} = \frac{1}{2}\,\frac{2\sqrt{u^3}}{3}$

$= \frac{\sqrt{u^3}}{3}\Big]_1^9 = \frac{\sqrt{9^3}}{3} - \frac{\sqrt{1^3}}{3} = \frac{27}{3} - \frac{1}{3} = \boxed{\frac{26}{3}}$

INTEGRALES TRIGONOMETRICAS

$\operatorname{Sen}^M x \cdot \cos^n x\,dx$

• Si las potencias de sen y cos son pares, use las identidades de angulos medios (o es par), si son impares aisle la potencia menor.

$\operatorname{Sen}^2\theta = \frac{1-\cos(2\theta)}{2} \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$

• Si la potencia del seno es impar, aisle sen (x)dx para hacer $u = \cos x$ recordando que $\operatorname{Sen}^2 x + \cos^2 x = 1$
• Si la potencia del coseno es impar, aisle cos (x) dx para para hacer $u = \operatorname{sen} x$ recordando que $\operatorname{Sen}^2 x + \cos^2 x = 1$

$\int_0^{\pi/2} \operatorname{Sen}^5\theta\,d\theta = \int_0^{\pi/2} \operatorname{Sen}^4\theta \cdot \operatorname{Sen}\theta\,d\theta = \int_0^{\pi/2} (1-\cos^2\theta) \cdot \operatorname{Sen}\theta\,d\theta$

$u = \cos x \qquad g(\pi/2) = \cos \pi/2$

$du = -\operatorname{Sen}\theta\,d\theta \qquad g(0) = \cos 0$

$$\int_{\cos 0}^{\cos \pi/2} (1-u^2)^2 \cdot (-du) = -\int_0^1 1-2u^2+u^4\, du$$

$$= \int_1^0 1\, du - 2\int_1^0 u^2 du + \int_1^0 u^4\, du$$

$$= u - 2\frac{u^3}{3} + \frac{u^5}{5}\Big|_1^0 = 0 - \frac{2(0)^3}{3} + \frac{(0)^5}{5} - \left[1 - \frac{2(1)^3}{3} + \frac{(1)^5}{5}\right]$$

$$= -\frac{1}{1} + \frac{2}{3} - \frac{1}{5}$$

$$= \frac{-15+10-3}{15} = \frac{8}{15} = \boxed{\frac{8}{15}}$$

$\tan^m x \cdot \sec^n x\, dx$

- Si la potencia de la secante es par use $\sec^2 x = 1 + \tan^2 x$ para hacer $u = \tan x$
- Si la potencia de la tangente es impar guarde un factor $\sec x$ para hacer $u = \tan x$

- Sea:

$*\int \text{sen}(mx) \cdot \text{sen}(nx)\, dx = \int \frac{1}{2}[\cos(mx-nx) - \cos(mx+nx)]$

$*\int \cos(mx) \cdot \cos(nx)\, dx = \int \frac{1}{2}[\cos(mx-nx) + \cos(mx+nx)]$

$*\int \cos(mx) \cdot \text{sen}(nx)\, dx = \int \frac{1}{2}[\text{sen}(mx+nx) - \text{sen}(mx-nx)]$

$*\int \text{sen}(mx) \cdot \cos(nx)\, dx = \int \frac{1}{2}[\text{sen}(mx+nx) + \text{sen}(mx-nx)]$

INTEGRACION POR PARTES

- Cuando usarla

* $f(x) \cdot g(x)$, con $f(x)$ y $g(x)$ de distinto tipo de funcion

* Integrales inversos "sen^{-1}, $\cos^{-1}$, $\tan^{-1}$, ..."

* $\int \ln x \, dx$

* $\int \sec^n x$, si n es impar

- Como usarla

* Utilizando LIATE para definir que funcion sera "U"

Logaritmicas
Inversas de T
Algebraicas
Trigonometricas
Exponenciales

* Defina cual sera "dv" y halle su integral "v"

- $\int f(x) \cdot g'(x) \cdot dx = f(x) \cdot g(x) - \int g(x) \cdot f'(x)$

$U = f(x) \qquad du = f'(x)$
$V = g(x) \qquad dv = g'(x)$

$$\int u\,dv = uv - \int v\,du \longrightarrow$$ Para Integrales Indefinidas

$$\int \frac{x}{1} \frac{\cos(5x)}{2} dx$$

L I A T E

$U = x \qquad dv = \cos(5x)$
$du = dx \qquad V = \frac{\operatorname{sen}(5x)}{5}$

$$= x \cdot \frac{\operatorname{sen}(5x)}{5} - \int \frac{\operatorname{sen}(5x)}{5}\,dx$$

$$= \frac{1}{5}\; x \cdot \operatorname{sen}(5x) - \frac{1}{5}\int \operatorname{sen}(5x)\,dx$$

$$= \frac{1}{5}\; x \cdot \operatorname{sen}(5x) - \frac{1}{5}\left(\frac{-\cos 5x}{5}\right)$$

$$= \frac{1}{5}\left[x \operatorname{sen}(5x) + \frac{\cos(5x)}{5}\right]$$

$$= \frac{1}{5}\left[\frac{5x \operatorname{sen}(5x) + \cos(5x)}{5}\right] + C$$

- $$\int_a^b f(x)\cdot g'(x)\,dx = f(x)\cdot g(x)\Big]_a^b - \int_a^b g(x)\cdot f'(x)\,dx$$

$u = f(x)$ $\quad$ $dv = g'(x)$

$du = f'(x)$ $\quad$ $v = g(x)$

$$\int_a^b u \cdot dv = u \cdot v\Big]_a^b - \int_a^b v \cdot du$$

$$\int_0^1 \underset{1}{\tan^{-1}}\, \underset{2}{dx}$$

$u = \tan^{-1}$ $\quad$ $v = x$

$du = \frac{1}{1+x^2}\,dx$ $\quad$ $dv = dx$

L $\overset{1}{\text{I}}$ $\overset{2}{\text{A}}$ T E

$$\int_0^1 \tan^{-1} dx = \tan^{-1}x \cdot x\Big]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= (1)\tan^{-1}(1) - \tan^{-1}(0) - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \frac{\pi}{4} - \underbrace{\int_0^1 \frac{x}{x^2+1}\,dx}_{(*)}$$

$$\int_0^1 \frac{x}{x^2+1}\,dx = \int_1^2 \frac{du}{2}\cdot\frac{1}{u} = \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$g(1) = 1+1 = 2$

$g(0) = 0+1 = 1$

$$= \frac{1}{2}\int_1^2 \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big]_1^2 = \frac{1}{2}\left[\ln|2| - \ln|1|\right]$$

$$= \boxed{\frac{\ln 2}{2}}$$

$$\boxed{\int_0^1 \tan^{-1}x\,dx = \frac{\pi}{4} - \frac{\ln 2}{2}}$$

SUSTITUCION TRIGONOMETRICA

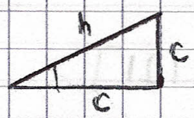
Se utiliza cuando hay potencias pares:

$\sqrt{a^2-x^2}$ ó $(a^2-x^2)^{m/n}$ siendo m y n iguales	$x = a\,\text{Sen}\,\theta$	$1-\text{Sen}^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$ ó $(a^2+x^2)^{m/n}$ siendo $m=n$	$x = a\tan\theta$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$ ó $(x^2-a^2)^{m/n}$ siendo $m=n$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

- Defina "x" $[g(x)]$
- Derive "x" "dx" $[g'(x)]$
- Remplaze
- Si es necesario halle los nuevos limites $g(b)$ y $g(a)$
- Use identidades para cancelar raices
- Opere
- Integre
- Remplace en la variable original si es necesario.

* Para volver a la variable original

1. Valla a la sustitución trigonometrica original "x" [g(x)]
2. Despeje la función (sen, cos, tan, ...)
3. Utilice el triangulo para hallar los valores de la hipotenusa y los catetos



4. Remplace de acuerdo a los valores hallados

$$\int_0^{0.6} \frac{x^2\,dx}{\sqrt{25\,(9/25 - x^2)}} = \int_0^{0.6} \frac{x^2\,dx}{5\,\sqrt{(9/25) - x^2}} = \frac{1}{5}\int_0^{0.6} \frac{x^2\,dx}{\sqrt{(9/25) - x^2}}$$

$x = \frac{3}{5} \cdot \operatorname{sen}\theta$ $\qquad a^2 = \frac{9}{25}$

$dx = \frac{3}{5}\cos\theta$ $\qquad a = \frac{3}{5}$

$g(b) \Rightarrow (0.6) = \frac{3}{5} \cdot \operatorname{sen}\theta$

$\operatorname{sen}\theta = \frac{3}{5} \cdot \frac{5}{3}$

$\boxed{\theta = \pi/2}$

$g(a) \Rightarrow 0 = \frac{3}{5} \cdot \operatorname{sen}\theta$

$\operatorname{sen}\theta = 0$

$\boxed{\theta = 0}$

$$\frac{1}{5}\int_0^{\pi/2} \frac{\frac{9}{25}\cdot \operatorname{sen}^2\theta}{\sqrt{\frac{9}{25} - \frac{9}{25}\operatorname{sen}^2\theta}} \cdot \frac{3}{5}\cdot\cos\theta = \frac{1}{5}\int_0^{\pi/2} \frac{\frac{9}{25}\cdot \operatorname{sen}^2\theta}{\cancel{\frac{3}{5}}\sqrt{1-\operatorname{sen}^2\theta}} \cdot \cancel{\frac{3}{5}}\cos\theta$$

$$\frac{1}{5}\int_0^{\pi/2} \frac{(9/25)\cdot \operatorname{sen}^2\theta}{\sqrt{\cos^2\theta}}\cos\theta = \frac{9}{125}\int_0^{\pi/2} \frac{\operatorname{sen}^2\theta}{|\cos\theta|}\cdot\cos\theta$$

$$= \frac{9}{125}\int_0^{\pi/2} \frac{\text{sen}^2\theta}{\cos\theta}\cdot\cos\theta = \frac{9}{125}\int_0^{\pi/2} \frac{1-\cos(2\theta)}{2}$$

$\downarrow$

$\cos\theta > 0$

$$= \frac{9}{125}\int_0^{\pi/2} \frac{1}{2} - \frac{\cos(2\theta)}{2} = \frac{9}{125}\cdot\frac{1}{2}\int_0^{\pi/2} 1 - \cos(2\theta)\, d\theta$$

$$= \frac{9}{250}\int_0^{\pi/2} d\theta - \int_0^{\pi/2} \cos 2\theta \, d\theta$$

$$= \frac{9}{250}\left[\theta - \frac{\text{sen}(2\theta)}{2}\right]\Big|_0^{\pi/2}$$

Aca podemos evaluar entre los limites $[0, \pi/2]$ o volver a la variable original y evaluar entre $[0 ; 0{,}6]$

(1)

$$= \frac{9}{250}\left[\theta - \frac{\text{Sen } 2\theta}{2}\right]_0^{\pi/2} = ((\pi/2)-0) - \left[\frac{2\,\text{Sen}\theta\cdot\cos\theta}{2}\right]$$

$$= \frac{\pi}{2} - \left[\text{sen}(\pi/2)\cdot\cos(\pi/2) - \text{sen}(0)\cdot\cos(0)\right]\left(\frac{9}{250}\right)$$

$$= \frac{\pi}{2} - \left[(0\cdot 1) - (0\cdot 1)\right]\left[\frac{9}{250}\right]$$

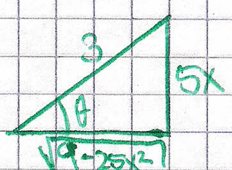
$$= \frac{9}{250}\cdot\frac{\pi}{2}$$

$$= \boxed{\frac{9\pi}{125}}$$

(2)

$$x = \frac{3}{5}\,\text{sen}\,\theta$$

$$\text{Sen}\,\theta = \frac{5X}{3}$$



$$(3)^2 = (5X)^2 + C^2$$

$$C = \sqrt{-25x^2 + 9}$$

$$C = \sqrt{9 - 25x^2}$$

$$\theta = \text{Sen}^{-1}\frac{5X}{3}$$

$$= \frac{9}{250}\left[\theta - \operatorname{Sen}\theta \cdot \cos\theta\right] = \frac{9}{250}\left[\operatorname{Sen}^{-1}\frac{5x}{3} - \left[\frac{5x}{3}\right]\left[\frac{\sqrt{9-25x^2}}{3}\right]\right]$$

$$= \frac{9}{250}\left[\operatorname{Sen}^{-1}\left(\frac{5x}{3}\right) - \left(\frac{5x}{3}\right)\left(\frac{\sqrt{9-25x^2}}{3}\right)\right]_0^{0.6}$$

$$= \frac{9}{250}\left[\operatorname{Sen}^{-1}(1) - (1)\sqrt{\frac{9-\frac{9}{25}\cdot 25}{9}} - \left[\operatorname{Sen}^{-1}(0) - (0)\cdot\frac{\sqrt{9-0}}{3}\right]\right]$$

$$= \frac{9}{250}\left[\frac{\pi}{2} - (1)\frac{0}{3} - 0 - 0\right]$$

$$= \frac{9}{250}\left[\frac{\pi}{2}\right]$$

$$= \boxed{\frac{9\pi}{125}}$$

INTEGRALES IMPROPIAS

Ejemplo: $\int_0^{+\infty} \frac{2x^2-x+4}{x^3+4x}$

Solucion

$f(x) = \frac{2x^2-x+4}{x^3+4x}$ no es continua en $x=0$

$$\int_0^1 \frac{2x^2-x+4}{x^3+4x}\,dx + \int_1^{+\infty} \frac{2x^2-x+4}{x^3+4x}\,dx$$

$$\lim_{b \to 0^+} \int_b^1 \frac{2x^2-x+4}{x(x^2+4)}\,dx + \lim_{c \to \infty} \int_1^c \frac{2x^2-x+4}{x(x^2+4)}\,dx$$

$$\int \frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$A=1$

Para $\frac{Bx+C}{x^2+4}$ Damos valores a x

$x=1$ $\quad 5/5 = 1 + (B+C)/5$ $\to$ Revisar

$x=-1$

Integral Segundo parcial

- Area entre dos curvas

1. Hallar puntos de interseccion, se igualan las ecuaciones y se hallan los limites de integracion

Vertical (x)

$$\int_a^b \frac{f(x)\text{superior} - f(x)\text{Inferior}}{dx}$$

Horizontal (y)

$$\int_a^b \frac{f(y)\text{Derecha} - f(y)\text{Izquierda}}{dy}$$

- Solidos de revolucion

1. Se rompe en base al eje de rotacion
2. Hallar limites de integracion

- Discos

$$\int_a^b \pi \cdot R^2 \, dx$$

$$R^2 = [f(x)\text{superior} - f(x)\text{inferior}]^2$$

- Arandela

$$\int_a^b \pi \, [R^2 - r^2] \, dx$$

$$R^2 = [\text{Eje de rotacion} - f(x)\text{superior}]^2$$

$$r^2 = [\text{Eje de rotacion} - f(x)\text{ Inferior}]^2$$

- Cascaron cilindrico

$$\int_a^b 2\pi \, x \cdot f(x) \, dx$$

$$2\pi \int_a^b x \cdot f(x) \, dx$$

- Curvas Parametricas $x = f(t)$, $y = g(t)$ $\alpha \le t \le \beta$

1. Si se puede, elimine el parametro
2. Halle el sentido dando valores a t

• Area

$$\int_{\alpha}^{\beta} g(t) \cdot f'(t)\, dt \quad \text{ó} \quad \int_{\beta}^{\alpha} g(t)\; f'(t)\, dt$$

• Longitud de Arco

$$\int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

- Coordenadas Polares

$x = r\cos\theta$ $\quad$ $\operatorname{sen}\theta = \frac{y}{r}$ $\quad$ $\cos\theta = \frac{x}{r}$

$y = r\operatorname{sen}\theta$

$\tan\theta = \frac{y}{x}$

$x^2 + y^2 = r^2$

* Figuras en polares

1. Circunferencia centrada en el polo (forma 1)

$r = f(\theta) = a$ ó $r = f(\theta) = -\theta$

$r = -2$
$r = 3$
...

2. Circunferencia centrada en el polo (forma 2)

$\to r_{max} = \text{Diametro}$

$r = \pm a\cos\theta$ ó $r = \pm a\operatorname{sen}\theta$ ó $r = \pm a\cos\theta \pm a\operatorname{sen}\theta$

Analisis de r maximo para hallar Diametro

3. Recta que pasa por el polo

$\theta = \alpha$ $\quad$ $\theta = -\pi/6$

4. Cardioides, con punto en el polo

Dar valores tipo circulo unidad $(0, \pi/2, \pi, 3\pi/2)$

$r = \pm a(1 \pm \cos\theta)$ o $r = \pm a(1 \pm \operatorname{sen}\theta)$

5. Rosas de petalos, que pasa por el polo

$r = \pm a \operatorname{sen}(n\theta)$ ó $r = \pm a \cos(n\theta)$

Si n es impar tiene n petalos

Si n es par tiene $2n$ petalos

* Analisis de r Maximo para graficar los petalos

- Area

$$A = \int_{\alpha}^{\beta} \frac{1}{2} (f(\theta))^2 \, d\theta, \quad f(\theta) = r$$

- Area entre dos polares

$$A = \int_{\alpha}^{\beta} \frac{1}{2} [f^2(\theta) - g^2(\theta)] \, d\theta$$

- Longitud de arco

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

- Conexion entre coordenadas polares y cartesianas

- Cartesianas a polares

$x = r\cos\theta$

$y = r\operatorname{sen}\theta$

$x^2 + y^2 = r^2$

$(x+1)^2 + (y+1)^2 = 2$

$(r\cos\theta + 1)^2 + (r\operatorname{sen}\theta + 1)^2 = 2$

$r^2 + 2r\cos\theta + 2r\operatorname{sen}\theta = 0$

$r(r + 2\cos\theta + 2\operatorname{sen}\theta) = 0$

$r_1 = 0$

$\boxed{r = -2\cos\theta - 2\operatorname{sen}\theta}$

- Polares o Cartesianas

$\tan\theta = \frac{y}{x}$ | $r = \pm\sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$, $x \neq 0$

- Trabajo. Momentos y centros de masa.

$F = m \cdot a$ Newton: Kg m/seg²

$F = m \cdot \frac{da^2}{dt^2}$ Dina: Lb · cm/seg²

- Trabajo con fuerza constante

$W = F \cdot d$ Julio: Newton · Metro

- Trabajo como integral

$W = \int_a^b F(x)\,dx$; $F(x)$ = Fuerza ; x = distancia

- Ley de Hooke

$F = K \cdot x$

- Valor promedio de una funcio

$$\frac{\int_a^b f(x)\,dx}{b-a}$$

- Ecuaciones diferenciales